Engineering Notes

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Failure Criterion for a Propellant of Spherical Solid Rocket Motor

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Introduction

PROPELLANT strength must be ensured in order to perform a rational design of a solid-propellant motor. The strength of a propellant is affected by its microstructure. Since the variance of the strength of composite propellants is "co" large, it becomes very important to analyze the mechanical properties of the propellant statistically. The flaw size of the propellant is finite, so the existence of a minimum strength can be expected. The strength estimated through the statistics of extremes was adopted as the design criterion for solid-propellant grains.

To examine the appropriateness of practical application of the criterion to actual rocket motors, two spherical solid rocket motors were utilized (Fig. 1). In one of them, the maximum strain was close to the one at which failure took place.

Statistical Analysis

The ultimate strength of the propellant can be expressed by a function of the maximum size of flaws. The cumulative distribution of the maximum diameter of flaws can be approximated by a logarithmic normal distribution function. Therefore,

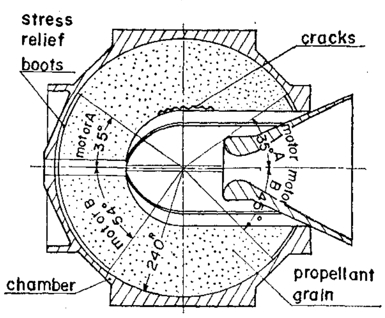

Fig. 1 Cross section of the spherical motor.

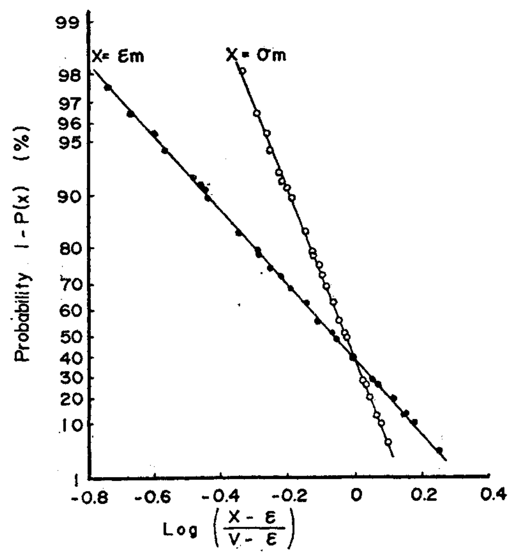

Fig. 2 Plot of observed strength data on probability coordinates.

the ultimate strength may be considered as a statistical variable. In the third asymptotic distribution of the smallest values, the failure probability, $P(x)$, for the load x may be expressed as follows:

$$P(x) = \exp[-(x-e)/v-e)]^k \qquad (1)$$

where e is the minimum strength, and v and k are characteristic and statistical parameters. The strain at the maximum stress in a uniaxial tensile test is adopted as the ultimate strength of the propellant because so many vacuoles in the specimen exist at the maximum stress.

The statistical parameters were obtained by the moment method proposed by Gumbel[1], and under the condition of five combinations of temperature and strain rate. In order to determine whether the third asymptotic distribution is applicable, the observed strength data were plotted on probability coordinates. The plotted points lay on a straight line, as seen in Fig. 2, so that the ultimate strength distribution of the composite propellant can be approximated well by the third asymptotic distribution. The curves of the mean and the statistical minimum values vs the reduced strain rate are shown in Fig. 3.

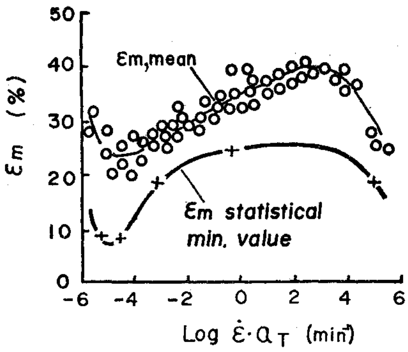

Fig. 3 Mean and statistical minimum values of ultimate strain vs reduced strain rate.

Presented as Paper 72-1088 at the AIAA/SAE 8th Joint Propulsion Specialist Conference, New Orleans, La., November 29–December 1, 1972; submitted November 8, 1972; revision received June 29, 1973.

Index categories: Solid and Hybrid Rocket Engines; Materials, Properties of; Thermal Stresses.

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Motor and Failure Mode

The propellant in the example motor is a composite solid consisting of CTPB binder, ammonium perchlorate and aluminum powder. The outside diameter of the spherical grain is 480 mm, and the geometry of the inner free volume of the grain is of the 8-slotted star shape. In order to reduce thermal strain in the grain, the surface between the grain and the chamber is unbounded. The difference between the two motors is in the depth of stress-relief boots. About a day after the cooling procedure was completed, fracture of the propellant occurred due to restrained thermal shrinkage along the inner surface of the grain. Many vacuoles, which appeared as white bands, were observed on the fillets of the inner bore of the grain. This means that the value of stress or strain was on the verge of the critical one at which the failure of the propellant took place. In order to estimate the strains due to thermal shrinkage at the inner fillet, three-dimensional photoelastic analysis[2,3] was carried out with respect to the variations of the depth of the stress-relief boots. The maximum strain concentration factors ($K_\varepsilon = \varepsilon/\varepsilon_0$) on the fillets of the motors A and B were estimated as 4.9 and 1.5, respectively (ε_0 is the tangential strain on the inner free surface of a long cylinder whose web-ratio is the same as the largest one in the spherical motor). It was known through the experimental analysis that the stress state along the fillets could almost be regarded as uniaxial.

Results

The restrained thermal strain of the spherical grains increases with decreasing grain temperature since the strain rate in the propellant should be proportional to the decreasing rate. The actual spherical rocket motors are cooled down at the rate of 0.5°C/hr. If the time-temperature shift factors had been already known, the curves of the ultimate strain could be easily given at any temperature by shifting the master curve along the temperature axis. In Fig. 4 the solid curve corresponds to the statistical line of the minimum strength, and the dotted solid line to the line of the observed minimum strength.

The tangential strains on the inner free surface of the grain whose outer surface is restrained to shrink can be expressed as

$$\varepsilon_p = (1+\mu)\left[(1-\mu)\frac{2K_\varepsilon M^2}{M^2(1-2\mu)+1} - 1\right](\alpha_p - \alpha_c)d\tau \quad (2)$$

where μ is Poisson's ratio of the propellant, dT is the temperature drop of the motor, M is the ratio of the outer to the inner diameters of the grain, α_p and α_c are thermal expansion coefficients of the propellant and the chamber, respectively. The straight lines in Fig. 4 show the strains in the two motors, which were calculated from Eq. (2). At about 30°C, the maximum strain of motor A (Fig. 1) reaches the master curve of the mean of ε_m. This corresponds to the fact that the white band (many vacuoles) appeared along the fillets of motor A when the motor was taken out from the cooling pit. On the other hand, any failure of the propellant of motor B did not occur when the maximum strain reached the line of the observed minimum strength at room temperature.

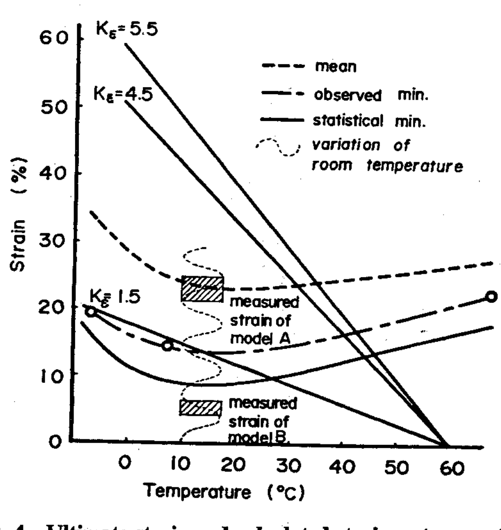

Fig. 4 Ultimate strain and calculated strain vs temperature.

If it is possible in a practical sense to adopt the statistical minimum value as the critical one for design of motors, this value will be most suitable because the safety of the propellant grain will be ensured statistically. It should be noticed in Fig. 3 that the statistical minimum values of the ultimate strength are close to the observed minimum values and are not so small in comparison with the mean of the ultimate strength. Therefore, the following relation is applicable as a practical design criterion

$$\varepsilon_{cr} = \varepsilon \quad (3)$$

where ε_{cr} is the design critical strain.

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[1] Gumbel, E. J., *Statistics of Extremes*, 4th printing, Columbia University Press, New York, 1967.

[2] Durelli, A. J. and Parks, V. J., "Photoelasticity Methods to Determine Stresses in Propellant Grain Models," *Experimental Mechanics*, Vol. 5, No. 2, Feb. 1965, p. 32.

[3] Sampson, R. C., "A Three-dimensional Photoelastic Method for Analysis of Differential Concentration Stresses," *Experimental Mechanics*, Vol. 3, No. 10, Oct. 1963, p. 225.

Conditions for Stability of an Ablating Symmetric Rolling Re-Entry Vehicle

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Nomenclature

A, B, C	= coefficients defined in Eq. (1)
$\bar{C}$	= a quantity defined in Eq. (8)
C_D	= drag coefficient
$C_{N\alpha}$	= normal force coefficient derivative
$C_{M\alpha}$	= pitching moment derivative, $\partial C_M/\partial\alpha$
$C_{Ma\alpha}$	= ablation induced pitching moment derivative
$C_{m\dot{\alpha}}$	= moment due to rate of angle-of-attack derivative
C_{mq}	= damping in pitch derivative
$C_{np\alpha}$	= Magnus moment derivative
D	= a coefficient defined in Eq. (1)
$G(s)$, $G_1(s)$, $G_2(s)$	= functions defined in Eq. (20)
g	= a "perturbation" vector
H	= altitude
I, I_x	= transverse and axial moments of inertia
k_a, k_T	= radii of gyration, defined in Eq. (1)
l	= a characteristic length
m	= mass
P	= normalized rate of roll, pl/V

Received February 28, 1973, revision received August 9, 1973.

Index categories: Aircraft Handling, Stability and Control; Entry Vehicle Dynamics and Control.

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